Half-lives of double β⁻-decay with two neutrinos*

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Nuclear double β⁻-decay with two neutrinos is an important decay mode for some unstable nuclei. Based on the available experimental data of nuclear double β⁻-decay, we propose that there is a law between the logarithm of double β⁻-decay half-lives and the reciprocal of the decay energy. The physics behind the law is discussed and it is found that this is associated with the universal properties of the weak interaction. This double β⁻-decay law is similar to the famous Geiger-Nuttall law of α-decay. The law is applied to predictions of the nuclear double β⁻-decay half-lives for six even-even nuclei from $Z = 84$ to $Z = 98$ and we found that ^{232}Th is very interesting for future experiments. The branching ratios between double β⁻-decay and α-decay are also estimated for the six even-even nuclei and this is useful for future experimental search of new emitters of double β⁻-decay.

Keywords: Double β⁻-decay with two neutrinos, Systematic law, Half-lives of heavy nuclei, The nucleus ^{232}Th.

I. INTRODUCTION

Researches on the β decay of nuclei lead to some important developments in both nuclear physics and particle physics. In order to explain the missing energy in β-decay, Pauli proposed in 1930s that there is a new particle, neutrino, who has carried away the missing energy. Fermi founded the basic theory of nuclear β-decay where the neutrino is involved in the theory although it is not discovered in 1930s. In 1950s, Lee and Yang proposed that parity can not conserve for a weak process such as β-decay [1]. Wu and her collaborators carried out the β-decay experiment with polarized ^{60}Co nuclei and confirmed that parity symmetry was broken in the β-decay process [2]. This brought to important impact to the development of physics. For double β-decay, it was first predicted by Goeppert-Mayer in 1935 [3] and it was later observed by the experiments of geochemistry, radiochemistry and nuclear physics [4–15]. Wu also gave good advice on the direct observation of rare double β-decay and finally the double β-decay of ^{82}Se was directly observed by Elliott, Hahn, and Moe [4]. To date, eleven nuclei are observed to have double β⁻-decay [10–13]. Although the data of double β-decay was accumulated, no simple law among the data was found. This is very different from the researches of other radioactivity such as α-decay and cluster-radioactivity where a unified formula between the half-lives and decay energies is proposed [16]. A new Geiger-Nuttall law [17] has been also proposed very recently and the influence of quantum numbers on α-decay half-lives is included in the formula [17, 18]. For numerical calculations of double β-decay half-lives, much of the theoretical research contains complicated calculations with long

codes in computers and can only be done by professional theoretical physicists. Of course some modern shell model calculations are still not available for heavy nuclei, such as ^{238}U, due to the limit of the computational ability of computers. Due to the computational complexity of double β-decay half-lives, double β-decay processes have not often been discussed in standard textbooks for undergraduate students and graduate students, although double β-decay has been clearly observed for many years. This is very different from both α-decay and complex cluster radioactivity, such as ^{14}C, where their half-lives can be easily calculated by simple formulas with three or four parameters. Therefore a simple and accurate way of calculating of double β-decay half-lives is needed. A simple and accurate way, and it is also useful for experimental physicists to estimate the decay half-live before an experiment of double β-decay and to analyze the data after the experiment.

II. NUMERICAL RESULTS AND DISCUSSION

Recently we make a systematic analysis on double β⁻-decay data and propose a new systematic law between the decay half-lives and the decay energy [19]. The law is simple and accurate for the ground state transition of double β⁻-decay between parent nuclei and daughter nuclei [19]. The law, without any extra adjustments, also works well for the double β⁻-decay between the ground state of parent nuclei and the first excited state 0^+ state of daughter nuclei [19]. Because the systematic law is proposed by analyzing the experimental data of the ground-state transitions of eleven even-even nuclei from ^{48}Ca to ^{238}U for double β⁻-decay, the effect of the weak interaction, the correction of the Coulomb interaction and the leading term of nuclear structure are taken into account in the law. The systematic law of double β-decay half-lives with two neutrinos is as follows [19]

$$\lg T_{1/2}(\text{Ey}) = (a - 2\lg(2\pi Z/137) + S)/Q_{2\beta}, \quad (1)$$

where $T_{1/2}$ is the double β^--decay half-life and its unit is Ey (10^{18} years), $Q_{2\beta}$ is the double β^--decay energy (MeV), Z is the charge number of the parent nucleus [19]. The first term of this equation corresponds to the impact of the weak interaction and the constant a equals to 5.843 by fitting the experimental double β^--decay data of ground state transitions for eleven even-even nuclei. The second term of equation (1) corresponds to the effect of the Coulomb potential for double β^--decay. The third term, however, is related to the shell effect and S is chosen to be $S = 2$ when the neutron number of the parent nucleus is a magic number, $S = 0$ when the neutron number of the parent nucleus is not a magic number.
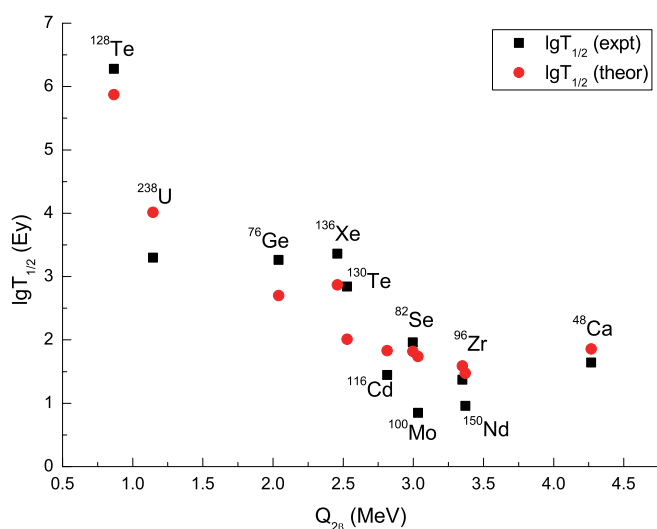

Fig. 1. (Color online)Variation of the logarithms of calculated and experimental double β-decay half-lives with the decay energy for eleven even-even nuclei from ^{48}Ca ($Z = 20$) to ^{238}U ($Z = 92$). It is interesting to note that the half-lives also depend on the charge number from Eq. (1). It is seen that the half-lives are very sensitive to the decay energies.

The variation of the logarithms of double β-decay half-lives with the decay energy is drawn in Fig. 1 for eleven even-even nuclei from ^{48}Ca ($Z = 20$) to ^{238}U ($Z = 92$). In Fig. 1, the X-axis is the decay energy of ground-state transitions and the Y-axis is the logarithm of double β-decay half-lives. Although the decay energy vary in a narrow range with several MeV, the half-lives varies in a very wide range from Ey (10^{18} years) to 10^6 Ey. This demonstrates the important effect of decay energies on half-lives. The higher decay energy usually leads to a shorter half-life for the nuclei on an isotopic chain. The calculated results with Eq. (1) reasonably agree with the available data and the number of adjusting parameters in Eq. (1) is the minimum, as compared to other theoretical calculations. The average deviation of half-lives of the eleven nuclei is a factor of three, which is good for a very simple formula of Eq. (1).

In the following context, we predict the double β-decay half-lives of six radioactive nuclei from ^{216}Po ($Z = 84$) to ^{256}Cf ($Z = 98$) by using Eq. (1). The calculated results of the six nuclei are listed in Table 1 and drawn in Fig. 2. For the
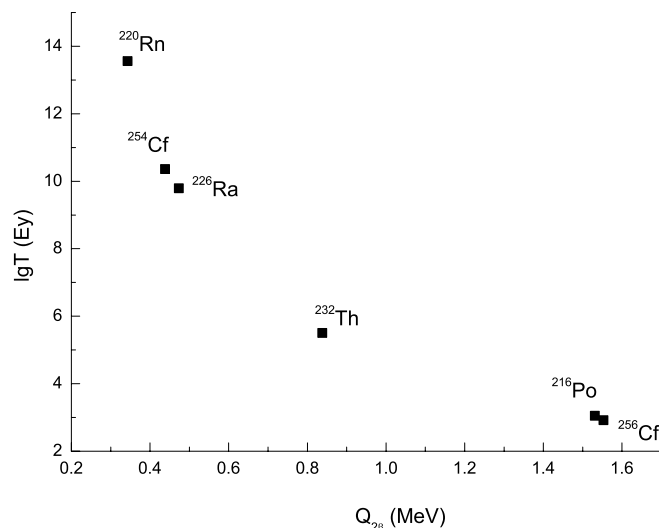

Fig. 2. Variation of the logarithms of calculated double β-decay half-lives with the decay energy for six even-even nuclei from ^{216}Po ($Z = 84$) to ^{256}Cf ($Z = 98$). It is interesting to note that the half-lives are also dependent on the charge numbers from Eq. (1). It is seen that the half-lives are very sensitive to the decay energies. The double β-decay half-life of ^{232}Th ($Z = 90$) is approximately 3×10^5 Ey and this shows that ^{232}Th is a very good candidate for experimental observation of double β-decay because it is rich in nature and it has a very long α-decay half-life ($T_{1/2}^{\alpha} = 1.4 \times 10^{10}$ years).

six nuclei, none of their neutron numbers are magic numbers, which means $S = 0$.

In Table 1, the first column denotes the parent nucleus and the second column represents the experimental double β-decay energy for the ground-state transition between the parent nucleus and the daughter nucleus where $Q_{2\beta} = M(A,Z) - M(A, Z - 2)$ [12, 13]. The calculated double β-decay half-lives ($T_{1/2}$(theor.)) are presented in the third column of the table and the units of the double β-decay half-lives are Ey (10^{18} years). We also listed the experimental α-decay half-lives ($T_{1/2}^{\alpha}$(expt.)) of the nuclei in column 4 for comparison with the data from references [12, 13]. The fifth column represents the branching ratio between double β^--decay and α-decay ($T_{1/2}^{\alpha}$(expt.)$/ T_{1/2}$(theor.)).

It is seen from Table 1 that the shortest double β-decay half-life is 8.36×10^2 Ey for ^{256}Cf and the longest is 3.64×10^{13} Ey for ^{220}Rn. They correspond to the highest double β-decay energy and the lowest double β-decay energy, respectively. This shows that the decay half-life strongly depends on the decay energy. Although ^{256}Cf and ^{216}Po have shorter half-lives for double β-decay, they are not suitable for future double β-decay experiments because their α-decay half-lives are too short. ^{220}Rn and ^{254}Cf are also not suitable for future search of new emitters of double β-decay because their double β-decay half-lives are too long and their α-decay half-lives are also too short. Only ^{226}Ra and ^{232}Th are interesting for future double β-decay experiments due to their longer α-decay half-lives. Especially ^{232}Th which has a very long α-decay half-life (1.4×10^{10} years) and shorter double β-

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TABLE 1. Theoretical double β^--decay half-lives ($T_{1/2}$(theor.)) for some heavy double β^--decay candidates where the experimental α-decay half-lives ($T^\alpha_{1/2}$(expt.)) are listed for comparison. The double β-decay energy and the branching ratio between the double β^--decay and the α-decay are also compared. The units of the double β-decay half-lives are Ey (10^{18} years). The experimental α-decay half-lives and double β-decay energies (MeV) in the table are from Refs. [12] and [13] respectively

Nuclei	$Q_{2\beta}$(MeV)	$T_{1/2}$(theor.)(Ey)	$T^\alpha_{1/2}$(expt.)	$T^\alpha_{1/2}$(expt.)$/T_{1/2}$(theor.)
^{216}Po	1.531	1.13×10^3	145 (ms)	4.07×10^{-30}
^{220}Rn	0.343	3.64×10^{13}	55.6 (s)	4.84×10^{-38}
^{226}Ra	0.473	6.19×10^9	1600 (y)	2.58×10^{-25}
^{232}Th [a]	0.838	3.19×10^5	1.4×10^{10} (y)	4.39×10^{-14}
^{254}Cf	0.438	2.30×10^{10}	60.5 (d)	7.21×10^{-30}
^{256}Cf	1.553# [b]	8.36×10^2	12.3 (m)	2.80×10^{-26}

[a] ^{232}Th will be very interesting for future experimental observation of double β-decay due to both its shorter double β-decay half-life and the longest α-decay half-life in six nuclei. ^{232}Th is also very rich in nature and this makes the experiment easier and cheaper as compared to other isotopes.

[b] For ^{256}Cf, the estimated mass is used for the calculation of the double β-decay energies (MeV) [12] and it is denoted with a # symbol.

decay half-life (3.19×10^5 Ey), this leads to its largest branching ratio between double β-decay and α-decay. This clearly shows that ^{232}Th is very interesting for future double β-decay experiments. ^{232}Th is also rich in nature and this makes the experiments easier and cheaper as compared to other isotopes.

The variation of the logarithms of double β-decay half-lives with the decay energy is also drawn in Fig. 2 for six even-even nuclei from ^{216}Po to ^{256}Cf. We can see again that the logarithms of double β-decay half-lives are inversely proportional to the decay energies. Usually a higher decay energy corresponds to a shorter double β-decay half-life and of course there is also a dependence on the charge number of parent nuclei.

Finally it is interesting to make a brief discussion about the law of double β-decay half-lives (equation (1)). For the systematic law of double β-decay half-lives, the first term in the law depends on the decay energy because the weak interaction is universal for natural decay processes and the total effect from the weak interaction is not very sensitive to the change of nucleon numbers such as charge number. This is different from that of α-decay where the total effect of the repulsive Coulomb potential is directly related to the charge number of the nuclei and α-decay occurs for ground states of medium and heavy nuclei [17, 18].

III. CONCLUSION

We presented a law for the calculations of double β-decay half-lives where the leading effects of the weak interaction, the Coulomb potential and the strong interaction are naturally taken into account. The law is an analytical formula for the half-lives of the complex double β-decay with two neutrinos and it can be easily introduced into textbooks due to its simplicity. The experimental data of double β-decay half-lives of ground-state transitions in eleven even-even nuclei are reasonably reproduced. We also discussed the universal behavior of the weak interaction through the formula of double β-decay half-lives. The law of double β-decay half-lives is as simple as that of the new Geiger-Nuttall law of α-decay. The half-lives of six double β-decay candidates with charge number from $Z = 84$ to $Z = 98$ are predicted and we found that ^{232}Th is very interesting for future experiments. The branching ratios between double β-decay and α-decay are estimated for six even-even nuclei, which are useful for future experiments as well.

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